

RESONANT CONVERSION OF STANDING ACOUSTIC OSCILLATIONS INTO ALFVÉN WAVES IN THE $\beta \sim 1$ REGION OF THE SOLAR ATMOSPHERE

D. Kuridze¹, T.V. Zaqarashvili^{1,2}, and B. Roberts³

1. Georgian National Astrophysical Observatory (Abastumani Astrophysical Observatory), Al. Kazbegi ave. 2a, 0160 Tbilisi, Georgia, E-mail: dato.k@genao.org, temury@genao.org

2. Departament de Física, Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain, E-mail: temury.zaqarashvili@uib.es

3. School of Mathematics and Statistics, University of St. Andrews, St. Andrews, Fife, KY16 9SS, UK, E-mail: bernie@mcs.st-and.ac.uk

ABSTRACT

We show that 5-minute acoustic oscillations may resonantly convert into Alfvén waves in the $\beta \sim 1$ region of the solar atmosphere. Considering the 5-minute oscillations as pumping standing acoustic waves oscillating along unperturbed vertical magnetic field, we find on solving the ideal MHD equations that amplitudes of Alfvén waves with twice the period and wavelength of acoustic waves exponentially grow in time when the sound and Alfvén speeds are equal, i.e. $c_s \approx v_A$. The region of the solar atmosphere where this equality takes place we call a *swing layer*. The amplified Alfvén waves may easily pass through the chromosphere and transition region carrying the energy of p-modes into the corona.

Key words: solar atmosphere; 5-minute oscillations; Alfvén waves.

1. INTRODUCTION

It is generally considered that solar 5-minute photospheric acoustic oscillations cannot penetrate into the upper regions due to the acoustic cutoff of stratified atmosphere. For the typical photospheric sound speed $c_s = 7.5$ km/s the cutoff frequency is 0.03 s⁻¹, which gives the cutoff period of 210 s (Roberts, 2004). This means that the sound waves with 5 min (~ 300 s) period are evanescent. Acoustic oscillations cannot penetrate into the corona also due to the sharp temperature gradient in the transition region. On the other hand, 3 and 5-minute intensity oscillations are intensively observed in the corona by the space satellites SOHO (Solar and Heliospheric Observatory) and TRACE (Transition Region and Coronal Explorer) (De Moortel, 2002). Re-

cently De Pontieu et al. (2005) have discussed how photospheric oscillations can be channelled into the corona through inclined magnetic fields. Another solution of this controversy is the conversion of acoustic oscillations into another wave mode, which may pass through chromosphere/transition region.

Recent two dimensional numerical simulations (Rosenthal et al., 2002; Bogdan et al., 2003) outlined the importance of $\beta \sim 1$ region in the solar atmosphere. They found the coupling of fast and slow magnetosonic waves at this particular region. Recent modelling of the plasma β in the solar atmosphere (Gary 2001, see Fig. 3 of that paper) shows that $v_A \sim c_s$, i.e. $\beta \sim 1$ (actually $\beta \sim 1.2$ for $\gamma = 5/3$), may takes place not only in lower chromosphere, but also at relatively low coronal heights (e.g., $\sim 1.2R_0$ from the surface, where R_0 is the solar radius). Also latest observations (Muglach et al., 2005) suggest the possible transformation of compressible wave energy into incompressible waves at $\beta \approx 1$ region of the solar atmosphere. Thus this particular region may be of importance due to conversion of compressible wave energy into incompressible Alfvén waves (or into MHD kink waves in thin photospheric magnetic tubes).

The coupling of propagating sound and Alfvén waves at $\beta \sim 1$ has been proposed by Zaqarashvili and Roberts (2002,2005). They found that a sound wave is nonlinearly coupled to the Alfvén wave with double the period and wavelength when the sound and Alfvén speeds are equal, i.e. $c_s \approx v_A$.

Ulrich (1996) has reported observations of Alfvén waves in the solar photosphere and lower chromosphere with substantial power at frequencies lower than the 5 minute oscillation. In the power spectrum of magnetic oscillations (Fig.3 in that paper) there is significant power at about 10 min.

Here we show that standing acoustic waves oscillating along uniform magnetic field lines effectively generate Alfvén waves with double their period and wavelength in the $\beta \sim 1$ regions of the solar atmosphere. The case of propagating waves is discussed in Zaqarashvili and Roberts (2002,2005).

2. STATEMENT OF THE PROBLEM AND DEVELOPMENTS

Consider fluid motions \mathbf{u} in a magnetised medium (with zero viscosity and infinite electrical conductivity), as described by the ideal MHD equations:

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{u} - \mathbf{B} \nabla \cdot \mathbf{u}, \quad (1)$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla \left[p + \frac{B^2}{8\pi} \right] + \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{4\pi}, \quad (2)$$

$$\frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla) \rho + \rho \nabla \cdot \mathbf{u} = 0. \quad (3)$$

$$p = p_0 \left(\frac{\rho}{\rho_0} \right)^\gamma, \quad (4)$$

where ρ is the medium density, p is the pressure, \mathbf{u} is the velocity, \mathbf{B} is the magnetic field and γ is the ratio of specific heats. For simplicity the stratification is neglected here, but we plan to take it into account in future.

Cartesian coordinate system is adopted with the z axis directed vertically upwards from the solar surface. Spatially inhomogeneous (along the x axis) magnetic field is directed along the z axis (see Fig.1), i.e.

$$\mathbf{B}_0 = (0, 0, B_z(x)). \quad (5)$$

Plasma pressure and density also are assumed to have x dependence, so they are expressed as $p_0(x)$ and $\rho_0(x)$ respectively. The magnetic field and pressure satisfy the transverse pressure balance condition

$$p_0(x) + \frac{B_z^2(x)}{8\pi} = \text{const.} \quad (6)$$

Plasma β is defined as

$$\beta = \frac{8\pi p_0(x)}{B_z^2(x)} = 2c_s^2/\gamma v_A^2(x), \quad (7)$$

where $c_s = \sqrt{\gamma p_0/\rho_0}$ and $v_A(x) = B_z/\sqrt{4\pi\rho_0}$ are the sound and Alfvén speeds respectively. Note that we suggest the temperature to be homogeneous so the sound speed does not depend on the x coordinate.

We consider wave propagation along the z axes (thus along the magnetic field) and wave polarisation in

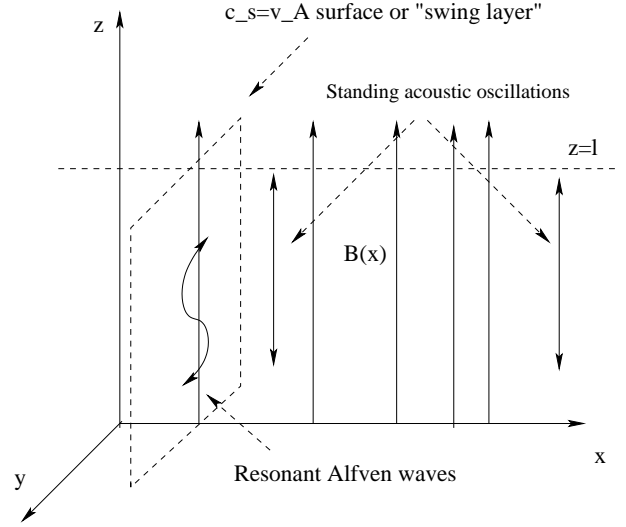


Figure 1. Schematic picture of the proposed process. The z coordinate is directed vertically upwards, while the x and y axes are parallel to the solar surface. The magnetic field and density are inhomogeneous along the x coordinate. Acoustic waves propagate up and down, i.e. along the z axis. Alfvén wave velocity oscillates along the y axis. Standing acoustic waves are resonantly converted into Alfvén waves near the region where $v_A \sim c_s$ called as swing layer.

yz plane. Then only sound and Alfvén waves arise. The velocity component of sound wave is polarised along the z axis and the velocity component of the Alfvén wave is polarised along the y -axis. In this case equations (1)-(3) take form

$$\frac{\partial b_y}{\partial t} + u_z \frac{\partial b_y}{\partial z} = -b_y \frac{\partial u_z}{\partial z} + B_z(x) \frac{\partial u_y}{\partial z}, \quad (8)$$

$$\rho \frac{\partial u_y}{\partial t} + \rho u_z \frac{\partial u_y}{\partial z} = \frac{B_z(x)}{4\pi} \frac{\partial b_y}{\partial z}, \quad (9)$$

$$\frac{\partial \rho}{\partial t} = -\rho \frac{\partial u_z}{\partial z} - u_z \frac{\partial \rho}{\partial z}, \quad (10)$$

$$\rho \frac{\partial u_z}{\partial t} + \rho u_z \frac{\partial u_z}{\partial z} = -\frac{\partial p}{\partial z} - \frac{\partial}{\partial z} \frac{b_y^2}{8\pi}, \quad (11)$$

$$\frac{\partial p}{\partial t} = -\gamma p \frac{\partial u_z}{\partial z} - u_z \frac{\partial p}{\partial z}, \quad (12)$$

where $p = p_0 + p_1$ and $\rho = \rho_0 + \rho_1$ denote the total (unperturbed plus perturbed) pressure and density, u_y and u_z are the velocity perturbations (of the Alfvén and sound waves, respectively), and b_y is the perturbation in the magnetic field. Note, that in these equations the x coordinate stands as a parameter.

Acoustic waves oscillate along the z axis, so that the velocity component has nodes at points $z = 0$ and $z = l$, i.e. $u_z = 0$ at $z = 0, l$, and thus we take

$$u_z = v(t) \sin(k_s z), \quad (13)$$

$$\rho_1 = \rho_1(t) \cos(k_s z), \quad (14)$$

where k_s is the wavenumber of sound wave such that

$$k_s l = \frac{2\pi l}{\lambda_s} = n\pi,$$

so

$$\frac{l}{\lambda_s} = n/2,$$

where $n = 1, 2, \dots$

We express the Alfvén wave components as

$$b_y = b(t) \cos(k_A z), \quad (15)$$

$$u_y = u(t) \sin(k_A z), \quad (16)$$

where k_A is the wavenumber of the Alfvén waves.

Substitution of expressions (13)-(16) into equations (8)-(12) and averaging with z over the distance $(0, l)$ leads to the cancelling of all nonlinear terms, which means that waves do not interact. However in particular case, when wave numbers k_s and k_A satisfy the conditions

$$k_s = 2k_A, \quad (17)$$

equations (8)-(12) take the form:

$$\frac{\partial b}{\partial t} = k_A B_0 u(t) - \frac{k_A}{2} v(t) b(t), \quad (18)$$

$$\frac{\partial u}{\partial t} = -\frac{k_A B_0}{4\pi\rho_0} \frac{b(t)}{1 - \frac{\rho_1(t)}{2\rho_0}} - \frac{k_A}{2} \frac{u(t)v(t)}{1 - \frac{\rho_1(t)}{2\rho_0}}, \quad (19)$$

$$\frac{\partial v}{\partial t} = \frac{k_s c_s^2}{\rho_0} \rho_1(t) + \frac{k_A}{8\pi\rho_0} b^2(t), \quad (20)$$

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 k_s v(t), \quad (21)$$

which means that the waves may interact as the nonlinear terms remain.

Substitution of u from equation (18) into equation (19) and neglecting of all third order terms leads to the second order differential equation

$$\begin{aligned} \frac{\partial^2 b}{\partial t^2} + k_A v \frac{\partial b}{\partial t} + \left[k_A^2 v_A^2 + \frac{k_A^2 v_A^2}{2\rho_0} \rho_1 + \frac{k_A}{2} \frac{\partial v}{\partial t} \right] b = \\ = 0. \end{aligned} \quad (22)$$

This equation describes the time evolution of Alfvén wave spatial Fourier harmonics expressed by (15)-(16) forced by standing acoustic waves.

In this equation, the first derivative with time can be avoided by substitution of function

$$b = \tilde{b}(t) e^{-\frac{k_A}{2} \int v dt},$$

which after dropping third order terms leads to

$$\frac{\partial^2 \tilde{b}}{\partial t^2} + \left[k_A^2 v_A^2 + \frac{k_A^2 v_A^2}{2\rho_0} \rho_1 \right] \tilde{b} = 0. \quad (23)$$

Equation (23) reflects the fact that the Alfvén speed is modified due to the density variation of standing acoustic wave. The similar equation for $\beta \gg 1$ was obtained by Zaqarashvili (2001). It is seen from this equation that the particular time dependence of density perturbation determines the type of equation and consequently its solutions. If we consider the initial amplitude of Alfvén waves smaller than the amplitude of acoustic waves, then the term with b^2 in equation (20) can be neglected. This means that the backreaction of Alfvén waves due to the ponderomotive force is small. Then the solution of equations (20)-(21) is just harmonic function of time

$$\rho_1 = \alpha \rho_0 \cos(\omega_s t), \quad (24)$$

where ω_s is the frequency of standing acoustic wave and $\alpha \ll 1$ is the relative amplitude. Here we consider the small amplitude acoustic waves $\alpha \ll 1$, so the nonlinear steepening due to the generation of higher harmonics is negligible. Then the substitution of expression (24) into equation (23) leads to the Mathieu equation

$$\frac{\partial^2 \tilde{b}}{\partial t^2} + k_A^2 v_A^2 \left[1 + \frac{\alpha}{2} \cos(\omega_s t) \right] \tilde{b} = 0. \quad (25)$$

The solution of this equation with frequency $\omega_s/2$ has an exponentially growing character, thus the main resonant solution occurs when (Zaqarashvili and Roberts, 2002; Shergelashvili et al., 2005)

$$\omega_A = v_A k_A = \frac{\omega_s}{2}, \quad (26)$$

where ω_A is the frequency of Alfvén waves. Since $k_A = k_s/2$, resonance takes place when

$$v_A = c_s. \quad (27)$$

Since the Alfvén speed $v_A(x) = B_z(x)/\sqrt{4\pi\rho_0(x)}$ is a function of the x coordinate, then this relation is satisfied at a particular location along the x axis (see Fig. 1). Therefore near this region the acoustic oscillations will be resonantly transformed into Alfvén waves. We call this region the *swing layer*, by analogy with mechanical swing interactions (see a similar consideration in Shergelashvili et al. (2005)).

Under condition (26) the solution of equation (25) is

$$\tilde{b}(t) = \tilde{b}_0 e^{\frac{|\alpha\omega_s|}{16} t} \left[\cos \frac{\omega_s}{2} t \mp \sin \frac{\omega_s}{2} t \right], \quad (28)$$

where $\tilde{b}_0 = \tilde{b}(0)$ and the phase sign depends on α ; it is $+$ for negative α and $-$ for positive α .

Note that the solution has a resonant character within the frequency interval

$$\left| \omega_A - \frac{\omega_s}{2} \right| < \left| \frac{\alpha\omega_s}{8} \right|. \quad (29)$$

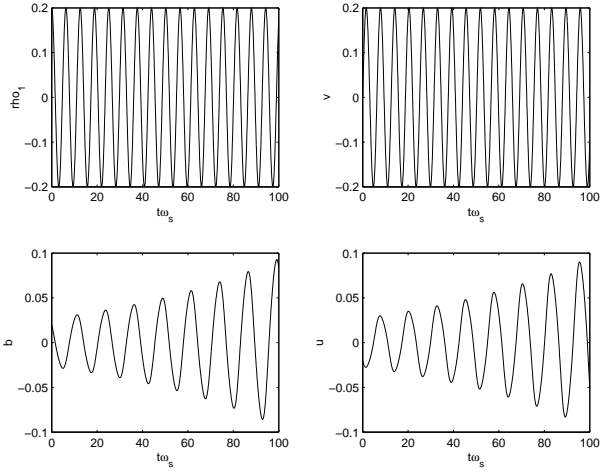


Figure 2. Numerical simulations of wave conversion. Upper panels show the density and velocity components of standing acoustic oscillations, while lower panels show the magnetic field and velocity components of Alfvén waves. Alfvén waves with twice the period of acoustic oscillations grow in time.

This expression can be rewritten as

$$\left| \frac{v_A}{c_s} - 1 \right| < \left| \frac{\alpha}{4} \right|. \quad (30)$$

Thus the thickness of the *swing layer* depends on the acoustic wave amplitude!

Therefore the acoustic oscillations are converted into Alfvén waves not only at the surface $v_A = c_s$ but also near that region, namely at

$$v_A = c_s \left(1 \pm \frac{\alpha}{4} \right). \quad (31)$$

Thus the resonant layer can be significantly wider for stronger amplitude acoustic oscillations.

Numerical solution of equations (18)-(21) (here the backreaction of Alfvén waves is again neglected) is presented on Fig.2. The amplification of Alfvén waves with double the period of acoustic oscillations is clearly seen.

3. CONCLUSIONS

We suggest that 3 and 5-minute acoustic oscillations in photosphere/chromosphere can be resonantly converted into Alfvén waves, or possibly into MHD kink waves in thin photospheric magnetic tubes, this process acting in the region of the solar atmosphere where $v_A \approx v_s$. Generated transversal waves may then propagate through the transition region into the corona, where they can deposit their energy back into density perturbations. The process can thus be of importance in coronal heating.

4. ACKNOWLEDGEMENTS

D. Kuridze and T. Zaqarashvili acknowledge the financial support from conference organisers.

REFERENCES

- Bogdan, T.J., Hansteen, M., Carlsson, V. et al. 2003, ApJ 599, 626
- De Moortel, I. Ireland, J., Hood, A. W. and Walsh, R.W. 2002, A&A **387**, L13
- De Pontieu, B., Erdelyi, R. and De Moortel, I. 2005, ApJ, 624, L61
- Gary, G.A. 2001, Solar Phys. **203**, 71
- Muglach, K., Hofmann, A. and Staude, J. 2005, A&A, 437, 1055
- Roberts, B. 2004, In Proc. SOHO 13 ‘Waves, Oscillations and Small-Scale Transient Events in the Solar Atmosphere: A Joint View from SOHO and TRACE’, Palma de Mallorca, Spain, (ESA SP-547), 1
- Rosenthal, C. S., Bogdan, T. J., Carlsson, M., Dorch, S. B. F., Hansteen, V., McIntosh, S. W., McMurry, A., Nordlund, . and Stein, R. F. 2002, ApJ 564, 508
- Shergelashvili, B.M., Zaqarashvili, T.V., Poedts, S. and Roberts, B. 2005, A&A 429, 767
- Ulrich, R.K. 1996, ApJ. **465**, 436
- Zaqarashvili, T.V., 2001, ApJ, 552, L81
- Zaqarashvili, T.V. and Roberts, B. 2002, Proc. 10th. European Solar Physics Meeting, “Solar Variability: ¿From Core to Outer Frontiers”, Prague, Czech Republic, 9-14 September 2002 (ESA SP-506, December 2002), 79
- Zaqarashvili, T.V. and Roberts, B. 2005, submitted